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Trajectory Estimation of Reentry Vehicles by Use of On-Line Input Estimator

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Trajectory estimation plays an important role in antitactical ballistic missile systems for tracking, guidance, and early warning. Model validation is a major challenge within the overall trajectory estimation problem. A novel on-line estimation approach in which an adaptive filter is used to identify the trajectory of a reentry vehicle in the terminal phase from single-source-measured radar data is presented. The proposed approach consists of an extended Kalman filter and an innovative recursive input estimator with an input detection. The model is formulated to identify an unmodeled acceleration input representing differences between the physical system and the mathematical model. A recursive least-squares estimator is provided to extract the magnitude of the unmodeled input and to offer a testing criterion for detection of the onset and the presence of the input. The estimated input is inserted into the extended Kalman filter if the input is detected. Numerical simulation demonstrates superior capabilities as measured by accuracy and robustness of the proposed method. In real-flight analysis, the adaptive filter also performs exceptionally well in terms of estimation and prediction. The recommended trajectory estimation method can support defense and tactical operations for antitactical ballistic missile warfare.

I. Introduction

IN antitactical ballistic missile (ATBM) warfare, on-line trajectory estimation of a reentry vehicle (RV) in radar tracking, missile guidance, and early-warning systems by use of measurements from a single radar is highly desired. Estimation of vehicle trajectories is typically investigated in postflight analysis to identify states and key parameters from available flight data measured by radar, satellite, and on-board sensors.[1–4] One of the major topics behind trajectory estimation problems relates to model validation, which focuses on the modeling errors between the physical system and the mathematical model. The model error is normally induced by simplification of assumptions, parameter uncertainty, maneuver and unpredictable external forces in flight, and other sources. On-line trajectory estimation in ATBM warfare is more difficult because all mathematical model quantities to be identified must be measurable by radar. Therefore a simplified model is desired, but it may induce a more serious model error problem. Stepwise regression methods play an important role in the determination of airplane model structure with both model orders and control derivatives.[5] It is, however, too complex to be implemented in on-line estimations. The on-line algorithm to be developed needs to operate well with a simplified model and to maintain a low sensitivity to the unavoidable model errors.

Estimation of a RV trajectory has received little attention. Chang et al. defined a filter for a maneuvering RV subject to a simplified dynamic model with maneuver forces in terms of related coefficients like drag force.[6] The augmented state vector in this filter contains position, velocity, and corresponding parameters for drag and maneuver forces and substitutes them into the extended Kalman filter (EKF) to perform the estimation work. The filter performance, however, is degraded if the maneuvering forces are absent. The dynamic model also confines the applications to maneuvering cases and is rarely applied to real flight. Manohar and Krishnan presented trajectory reconstruction of a rocket by using a differential corrections method.[7] The governing equations for estimation contain an unmodeled acceleration term to take care of all forces acting on the vehicle other than those from aerodynamic, propulsive, and gravitational sources. Trajectory states and unmodeled acceleration

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values are estimated simultaneously. The required data, nevertheless, are beyond what a single radar can provide. A simple model with the unmodeled acceleration inputs seems applicable for on-line trajectory estimation if a recursive determination of inputs is well defined.

Input estimation by batched least-squares methods has been successfully applied in tracking problems,[8–10] initial leveling of strapdown inertial navigation systems,[11,12] and inverse heat conduction problems.[13,14] The utilization of such a methodology leads to a new filtering approach based on the EKF with a recursive input estimator to execute on-line trajectory estimation for a RV. This new filtering strategy is the main focus of this paper.

This paper reformulates a simple model for a RV and presents an adaptive filter capable of providing accurate and fast trajectory estimation with low sensitivity to model errors. The proposed adaptive filter consists of the EKF and a recursive least-squares input estimator with an input detection criterion. The input estimator proposed herein can continuously estimate unknown acceleration inputs and substitute the estimated results into the EKF to identify the RV's position and velocity if the detection criterion is satisfied. Simulation results of a maneuvering RV indicate that use of the proposed method can provide significant improvement of accuracy and with low sensitivity compared with the conventional EKF. The estimation approach is evaluated with actual flight test data measured by a single radar from a test that further validates the methodology. Furthermore, from the data reconstruction, the recommended trajectory estimation can provide insight into the RV behavior and possibly facilitate analysis involving the ATBM strategy.

The remaining sections of this paper are organized as follows. Section II formulates the RV model with some assumptions and the EKF formation dedicated to the estimation problem. In Sec. III, we derive the adaptive filter with a novel input estimation, input detection, and the EKF for the RV. Section IV presents the simulation study and real-flight analysis to demonstrate the effectiveness and the robustness to the model errors and sample size. Finally, conclusive remarks are made in Sec. V.

II. Dynamic Model

The RV flies over several hundred kilometers along a ballistic trajectory above the rotating Earth. Constructing a mathematical model for this system with some simplifying assumptions to describe its motion is the first step for identification and analysis. Consider a vehicle in the reentry phase over a flat, nonrotating Earth with constant gravity, as illustrated in Fig. 1. The distance that the RV travels in this phase is shorter than the distance traveled before reentry. Consider the RV as a point mass with constant weight following a ballistic trajectory in which two types of significant forces, drag and gravity, act on the RV. Extra forces are induced by model error when assumptions are violated or the RV undertakes a maneuver. The effects can be grouped into an unmodeled acceleration in each axis. The RV model in radar coordinates (O_R, X_R, Y_R, Z_R) centered at the radar site can be expressed as[8,15]

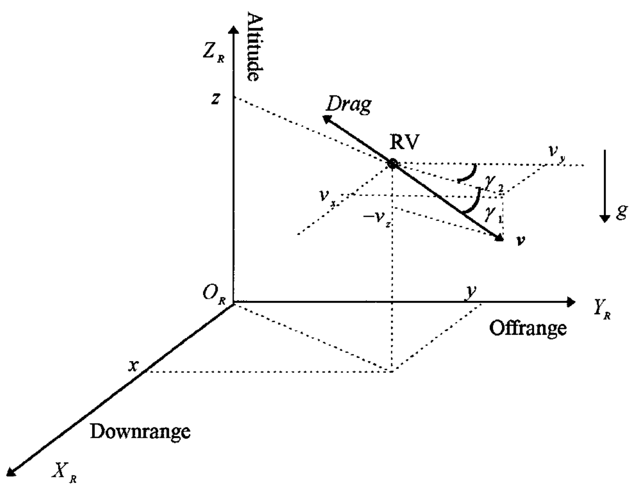

Fig. 1 Tactical ballistic missile geometry.

$$\dot{v}_x = -(\rho v^2/2\beta)g\cos\gamma_1\sin\gamma_2 + u_4 \tag{1}$$

$$\dot{v}_y = -(\rho v^2/2\beta)g\cos\gamma_1\cos\gamma_2 + u_5 \tag{2}$$

$$\dot{v}_z = (\rho v^2/2\beta)g\sin\gamma_1 - g + u_6 \tag{3}$$

where v_x, v_y, and v_z denote velocity components along X_R, Y_R, and Z_R, respectively; u_4, u_5, and u_6 are unmodeled accelerations generated by the model errors along X_R, Y_R, and Z_R, respectively; and ρ stands for air density. γ_1, γ_2, and ballistic coefficient β are defined as

$$\gamma_1 = \tan^{-1}\left(-\frac{v_z}{\sqrt{v_z^2+v_y^2}}\right), \qquad \gamma_2 = \tan^{-1}\left(\frac{v_x}{v_y}\right), \qquad \beta = \frac{W}{SC_{D0}}$$

where S, W, and C_{D0} represent the reference area, weight, and zero-lift drag coefficient, respectively. In a previous reference the constant drag model and constant β was investigated.[6]

The air density is a function of altitude and should be considered as such in Eqs. (1–3) because the altitude dramatically changes in flight over 100 km. An approximate model of air density in pounds per cubic feet as a function of altitude in feet is[15]

$$\rho = 0.002378e^{-z/30000} \qquad \text{for} \qquad z < 30{,}000\,\text{ft} \tag{4}$$

$$\rho = 0.0034e^{-z/22000} \qquad \text{for} \qquad z \geq 30{,}000\,\text{ft} \tag{5}$$

Let the state vector be

$$X = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T = [x \quad y \quad z \quad v_x \quad v_y \quad v_z]^T \tag{6}$$

where x, y, and z are the RV positions in X_R, Y_R, and Z_R, and respectively. The nonlinear state equation can be written as

$$\dot{X} = F(X) + \varphi u + I_{6\times 6}\zeta \tag{7}$$

In Eq. (7), ζ stands for the process noise vector with variance Q, I denotes the identity matrix, and u expresses the input vector originating from unmodeled accelerations:

$$F(X) = \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ -(\rho/2\beta)\left(x_4^2+x_5^2+x_6^2\right)g\cos\gamma_1\sin\gamma_2 \\ -(\rho/2\beta)\left(x_4^2+x_5^2+x_6^2\right)g\cos\gamma_1\cos\gamma_2 \\ -(\rho/2\beta)\left(x_4^2+x_5^2+x_6^2\right)g\sin\gamma_1 - g \end{bmatrix}$$

$$\varphi = \begin{bmatrix} 0_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & I_{3\times 3} \end{bmatrix}, \qquad u = [0 \quad 0 \quad 0 \quad u_4 \quad u_5 \quad u_6]^T$$

For conventional air defense missile systems, the ground radar is the major instrument for detecting the RV. The measurement equation is then

$$Z = HX + \varepsilon \tag{8}$$

where ε expresses the measurement noise vector and H denotes the response distribution matrix. For this analysis, $H = I$. Equations (7) and (8) form the dynamic equations for the vehicle during reentry. Once all states at a specific time are precisely known, the trajectory is reconstructed with a fixed-point smoother and an n-step predictor.[16]

Let ε be white and normally distributed noise with zero mean and variance R. The predicted and updated states vectors of the EKF from $t = n\Delta t$ to $t = (n+1)\Delta t$, $n = 0, 1, 2, \ldots$, under input vector u_n at $t = n\Delta t$ are given by[17]

$$\hat{X}_{n+1/n} = \phi_n\hat{X}_{n/n} + \varphi u_n \tag{9}$$

$$\hat{X}_{n+1/n+1} = \hat{X}_{n+1/n} + K_{n+1}(Z_{n+1} - H\hat{X}_{n+1/n}) \tag{10}$$

where Δt is the sampling period, Z_{n+1} denotes radar measurements at $t = (n+1)\Delta t$, and the transition matrix

$$\phi_n = I_{6\times 6} + \left.\frac{\partial F(\boldsymbol{X})}{\partial \boldsymbol{X}}\right|_{X=\hat{X}_{n/n}} \Delta t$$

Details of the linearization of $F(\boldsymbol{X})$ are given in the Appendix. The Kalman gain K_{n+1} and covariance matrices $P_{n+1/n}$ and $P_{n+1/n+1}$ are

$$K_{n+1} = P_{n+1/n}H^T\left(HP_{n+1/n}H^T + R\right)^{-1} \tag{11}$$

$$P_{n+1/n} = \phi_n P_{n/n}\phi_n^t + \Gamma Q \Gamma \tag{12}$$

$$P_{n+1/n+1} = (I - K_{n+1}H)P_{n+1/n} \tag{13}$$

where $\Gamma = I_{6\times 6}\Delta t$. The EKF will typically converge with long-time propagation if $\boldsymbol{u}_n$ is omitted in Eq. (9). However, a long convergence time is unacceptable for missile defense applications, which require rapid reaction within just a few minutes. Therefore an algorithm must be developed to estimate the input acceleration and achieve a rapid and accurate trajectory estimation that yields some desired level of accuracy.

III. Adaptive Filtering with Input Estimation

For a circumstance in which the input terms $\boldsymbol{u}_n$ in Eq. (9) remain unknown and the onset is unpredictable, this section presents an adaptive filter scheme that consists of the EKF and a recursive least-squares estimator for the input with an unmodeled acceleration detection criterion. Adaptive refers to a situation in which the filter sequentially adjusts to an input based on measurements. The input estimator attempts to estimate $\boldsymbol{u}_n$ by using radar measurements. Moreover, the estimated input joins the EKF if it satisfies the detection criterion.

A. Input Estimation

Let $\bar{X}_{n+1/n}$ and $\bar{X}_{n+1/n+1}$ denote the predicted and the updated states, respectively, for the EKF with no input at $t = (n+1)\Delta t$. For simplicity, denote $\hat{X}_{n+1} = \hat{X}_{n+1/n+1}$ and $\bar{X}_{n+1} = \bar{X}_{n+1/n+1}$, and let $M_{n+1} = (I - K_{n+1}H)\phi_n$, $N_{n+1} = (I - K_{n+1}H)\varphi$. The updated state can be represented by

$$\bar{X}_{n+l} = \left(\prod_{i=n+1}^{n+l} M_i\right)\bar{X}_n + \sum_{j=1}^{l-1}\left(\prod_{i=1+j}^{l} M_{n+i}\right)K_{n+j}Z_{n+j} + K_{n+l}Z_{n+l}, \qquad l = 0, 1, 2, \ldots \tag{14}$$

Assume that the abrupt deterministic inputs are applied during $k\Delta t \le t \le (k+s)\Delta t$:

$$\boldsymbol{u} = \begin{cases} 0, & t < k\Delta t, \quad t > (k+s)\Delta t, \quad k, s > 0 \\ \boldsymbol{u}_{k+l}, & k\Delta t \le t \le (k+s)\Delta t, \quad l = 0, 1, 2, \ldots, s \end{cases} \tag{15}$$

where $\boldsymbol{u}_{k+l}$ is a constant vector over the sampling interval. Similarly, the updated state vector in the EKF formation with input can be expressed as

$$\hat{X}_{k+l} = \left(\prod_{i=k+1}^{k+l} M_i\right)\hat{X}_k + \sum_{j=1}^{l-1}\left(\prod_{i=1+j}^{l} M_{k+i}\right)(K_{k+j}Z_{k+j} + N_{k+j}\boldsymbol{u}_{k+j-1}) + K_{k+l}Z_{k+l} + N_{k+l}\boldsymbol{u}_{k+l-1} \tag{16}$$

The EKF formulation with no input and with an input yields the same results during, $t \le k\Delta t$, so that $\hat{X}_k = \bar{X}_k$. The difference induced by the abrupt inputs between these two formations during $k\Delta t \le t \le (k+s)\Delta t$ can then be written as

$$\Delta \boldsymbol{X}_{k+l} = \hat{\boldsymbol{X}}_{k+l} - \bar{\boldsymbol{X}}_{k+l} = M_{k+l}\Delta\boldsymbol{X}_{k+l-1} + N_{k+l}\boldsymbol{u}_{k+l-1}$$

Define the measurement residual for the EKF formation without inputs to be $\bar{Z}_{k+l} = Z_{k+l} - H\bar{X}_{k+l}$. Therefore the recursive least-squares input estimator can be expressed by[12,18,19]

$$\hat{\boldsymbol{u}}_{k+l-1} = \hat{\boldsymbol{u}}_{k+l-2} + G_{k+l}(\hat{Y}_{k+l} - \Phi_{k+l}\hat{\boldsymbol{u}}_{k+l-2}) \tag{17}$$

where

$$\hat{Y}_{k+l} = \bar{Z}_{k+l} - H\sum_{j=1}^{l-1}\left(\prod_{i=1+j}^{l} M_{k+i}\right)N_{k+j}\hat{\boldsymbol{u}}_{k+j-1}$$

$$= \bar{Z}_{k+l} - HM_{k+l}\Delta\hat{\boldsymbol{X}}_{k+l-1}$$

$$\Delta\hat{\boldsymbol{X}}_{k+l-1} = \Delta\boldsymbol{X}_{k+l-1}|_{u=\hat{u}} = M_{k+l}\Delta\hat{\boldsymbol{X}}_{k+l-2} + N_{k+l}\hat{\boldsymbol{u}}_{k+l-2}$$

$$\Phi_{k+l} = HN_{k+l}$$

and the gain G_i and variance of $\hat{\boldsymbol{u}}_i$, V_i are

$$G_{k+l} = V_{k+l-1}\Phi_{k+l}\xi^{-1}$$

$$V_{k+l-1} = V_{k+l-2} - V_{k+l-2}\Phi_{k+l}^T \times\left[\Phi_{k+l}V_{k+l-2}\Phi_{k+l}^T + \xi\right]^{-1}\Phi_{k+l}V_{k+l-2}$$

where ξ is the variance of the measurement residual for the EKF with input. The innovated measurement $\hat{Y}_{k+l}$ contains only partial information in Z_{k+l}, subsequently leading to a biased input estimator. However, this biased estimator still functions elaborately.

B. Input Detection Criterion

In Eq. (15), k and s represent the starting and the stopping indices of the system input, respectively. As mentioned in Sec. II, the system input onset is unpredictable and cannot be assumed in advance. A testing method is presented in this section to determine k and s.

According to the central limit theorem,[20] we have

$$\hat{\boldsymbol{u}}_i \to N(\boldsymbol{u}_{bi}, V_{ii}), \qquad i = 4, 5, 6 \text{ as } t \to \infty$$

where $N(\boldsymbol{u}_{bi}, V_{ii})$ represents the normal distribution with mean $\boldsymbol{u}_{bi}$ and variance V_{ii} of $\hat{\boldsymbol{u}}_i$ and $\boldsymbol{u}_b$ denotes a certain value that corresponds to the true input $\boldsymbol{u}_i$ and equals zero if $\boldsymbol{u}_i$ does not exist. Therefore the test for detection of input is[20]

$$\left|\hat{\boldsymbol{u}}_i\big/\sqrt{V_{ii}}\right| > t_{st} \text{ existence of } \boldsymbol{u}_i, \qquad \text{for} \qquad i = 4, 5, 6 \tag{18}$$

Otherwise $\boldsymbol{u}_i$ is absent, where $[-t_{st}, t_{st}]$ is the confidence interval that can be determined by examining the cumulative normal distribution table for a certain preset confidence level α. Figure 2 illustrates the mechanism of the proposed scheme.

C. Adaptive Filtering

When the adjusted on-line input is used, the predicted and updated states for the adaptive filter at time interval $k\Delta t \le t \le (k+s)\Delta t$ are

$$\boldsymbol{X}^v_{k+l+1/k+l} = \phi_{k+l}\boldsymbol{X}^v_{k+l/k+l} + \varphi\hat{\boldsymbol{u}}_{k+l-1} \tag{19}$$

$$\boldsymbol{X}^v_{k+l+1/k+l+1} = \boldsymbol{X}^v_{k+l+1/k+l} + K^v_{k+l+1}\left(Z_{k+l+1} - H\boldsymbol{X}^v_{k+l+1/k+l}\right) \tag{20}$$

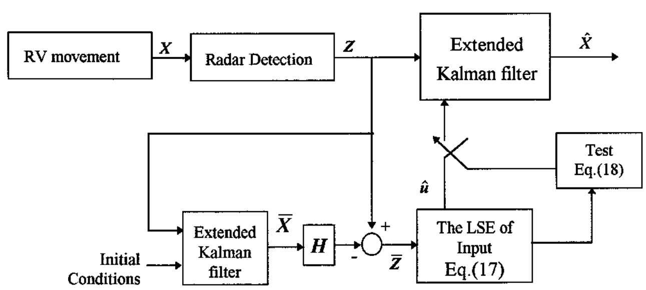

Fig. 2 Mechanism of the proposed adaptive filter scheme, LSE-least-squares estimate.

The Kalman gain becomes

$$K^v_{k+l+1} = P^v_{k+l+1/k+l} H^T \left(H P^v_{k+l+1/k+l} H^T + R \right)^{-1} \tag{21}$$

where the covariance matrices of the adaptive filter at $k\Delta t \le t \le (k+s)\Delta t$ are

$$P^v_{k+l+1/k+l} = P_{k+l+1/k+l} + \phi_{k+1} L_{k+l+1} \phi^T_{k+l} + \varphi V_{k+l} \varphi^T$$

$$= P_{k+l+1/k+l} + P'_{k+l+1/k+l} \tag{22}$$

$$P^v_{k+l+1/k+l+1} = \left(I - K^v_{k+l+1} H \right) P^v_{k+l+1/k+l} \tag{23}$$

where

$$P'_{k+l+1/k+l} = \phi_{k+l} L_{k+l+1} \phi^T_{k+l} + \varphi V_{k+l} \varphi^T$$

$$L_{k+l+1} = \sum_{j=1}^{l} \left(\prod_{i=1+j}^{l+1} M_{k+i-1} \right) N_{k+j} V_{k+j-1} N^T_{k+j}$$

$$\times \left(\prod_{i=1+j}^{l+1} M^T_{k+i-1} \right) = M_{k+l} L_{k+l} M^T_{k+l}$$

$P'_{k+l+1/k+l}$ represents the increment in covariance introduced by $\hat{\boldsymbol{u}}_i$; $i = k, k+1, \ldots, k+l$. It is easy to prove that $P'_{k+l+1/k+l}$ is positive definite, implying that when $\hat{\boldsymbol{u}}_i$, $i = k, k+1, \ldots, k+1$ is introduced to reduce the state estimation errors, the covariance of estimated states must increase.

For time beyond the interval $t < k\Delta t$ and $t > (k+s)\Delta t$, state estimation can be also based upon the original EKF. Note that the initial states and the covariance matrices at $t > (k+s)\Delta t$ are reinitiated by $\boldsymbol{X}^v_{k+s/k+s}$ and $P^v_{k+s/k+s}$.

IV. Case Studies

This section presents two cases to evaluate the effectiveness of the proposed adaptive filter scheme indicated in Fig. 2. These two cases utilize simulation data and real-flight data under confidence levels of $\alpha = 95\%$. The first case compares the proposed adaptive filter with the EKF without input estimator by use of the simulated data from Eq. (7) with a constant ballistic coefficient for a maneuvering RV. The second case investigates in an off-line fashion real-flight data obtained by a precision radar in a RV flight test.

A. Case 1: Maneuvering RV Analysis

The proposed method was validated with simulation data in this case. Consider an RV reentering along a ballistic trajectory that is simulated from Eq. (7) with $\beta = 2440$ kg/m^2 and maneuvering accelerations of $4\,g$, $2\,g$, and $-2\,g$ in X_R, Y_R, and Z_R, respectively, during $20 \le t \le 25$ s. β was held constant in simulation and identification. The first measured position and velocity are adopted as the initial values. The simulated trajectory with standard normally distributed measurement and process noises, $Q = I$ and $R = I$, are displayed in Figs. 3 and 4. Let the initial value of each V_{ii}, $i = 4, 5, 6$, be 10. The plot of V_{ii} is given in Fig. 5, which displays the fast convergence of input estimation. Figure 6 compares the position estimation errors of the proposed scheme with those of the EKF. The proposed scheme limits the estimation errors in position to within ± 4 m, which is much better than the ± 2000 m associated with the EKF. For the estimation in velocity, as indicated in Fig. 7, the errors produced by the proposed scheme are bounded by ± 6 m/s and remain much lower than those for the EKF, which is up to ± 200 m/s.

Table 1 summarizes the rms errors over the total flight time, maximum errors, and the steady errors at the impact point of these two methods. The rms errors made by the proposed method are much lower than those made by the EKF. Accuracy, roughly represented by the steady errors, is also much better than that obtained with

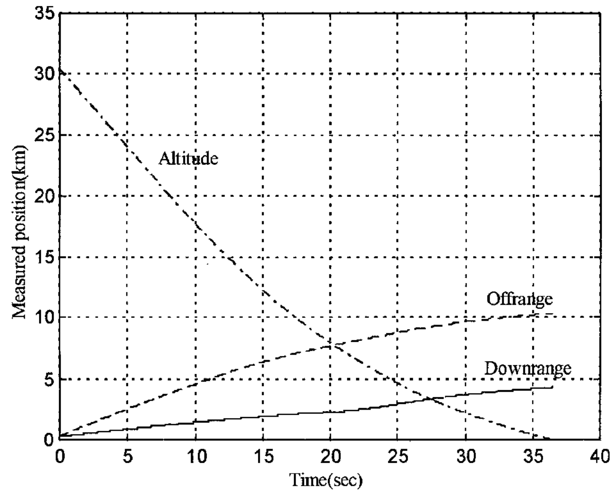

Fig. 3 Simulated RV position.

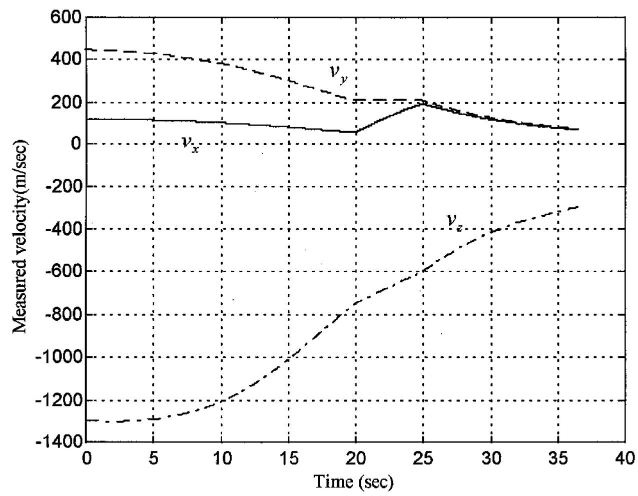

Fig. 4 Simulated RV velocity.

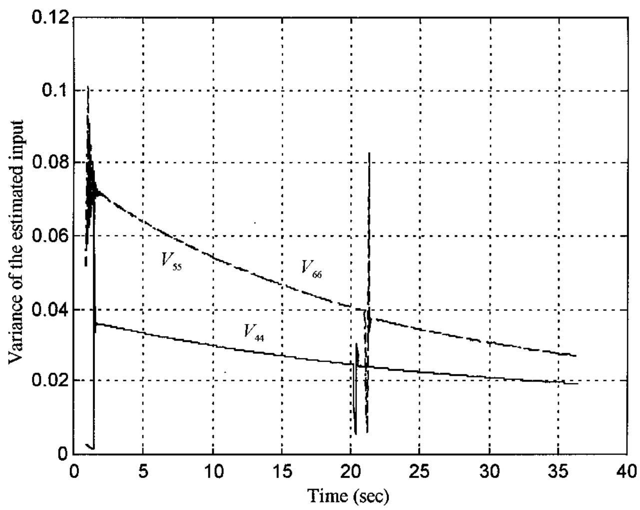

Fig. 5 Variances of the estimated input.

the EKF. The effectiveness of the proposed scheme is definitely verified.

As mentioned at the beginning of this subsection, the ballistic coefficient is assumed constant but is actually a function of Mach number, angle of attack, angle of sideslip, etc. The zero-lift drag coefficient C_{D0} within β is unknown with respect to the air defense missile and is arbitrarily assigned during estimation. Therefore tolerence of β uncertainty is highly desired. Table 2 lists the rms errors of the two methods under the situations of $\beta = 1000$, 4880,

Table 1 State estimation errors[a] for β = 2440 kg/m^2

States	Adaptive filter rms	Adaptive filter Maximum	Adaptive filter Steady	EKF rms	EKF Maximum	EKF Steady
x	0.25	0.73	0.016	337.9	710.13	11.51
y	0.62	1.5	0.073	376.55	718.03	−71.07
z	1.58	3.49	0.45	1.01×10^3	1.94×10^3	783.02
v_x	0.92	5.46	0.74	86.42	194.49	−4.59
v_y	0.55	3.18	0.45	83.15	173.08	22.87
v_z	1.06	5.48	−0.71	218.89	442.08	153.31

[a]Position and velocity are given in meters and meters per second, respectively.

Table 2 Root-mean-square estimation errors[a] for different β

States	Adaptive filter $\beta = 1{,}000$	Adaptive filter $\beta = 4{,}880$	Adaptive filter $\beta = 24{,}400$	EKF $\beta = 1{,}000$	EKF $\beta = 4{,}880$	EKF $\beta = 24{,}400$
x	0.45	0.21	0.23	1.02×10^3	173	84.14
y	1.01	0.52	0.45	785.18	333.65	489.3
z	2.75	1.98	1.05	213×10^3	1.01×10^3	1.55×10^3
v_x	0.96	0.94	1.13	234.77	46.33	33
v_y	0.99	0.49	0.39	149.09	68.17	91.37
v_z	2.39	3.41	0.78	464.38	167.45	267.5

[a]Position and velocity are given in meters and meters per second, respectively.

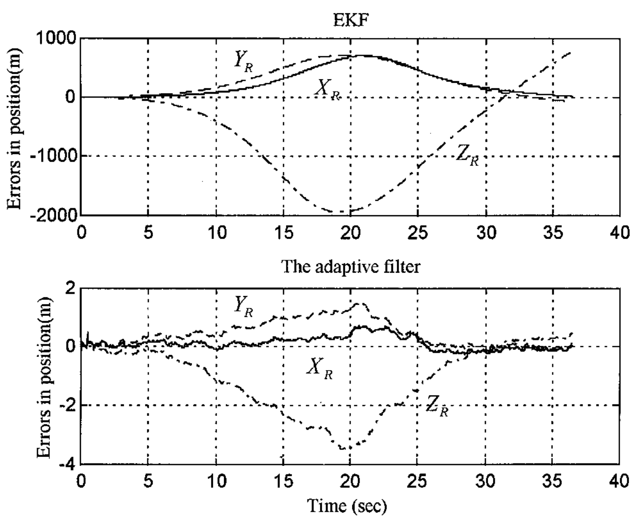

Fig. 6 Estimation errors in position.

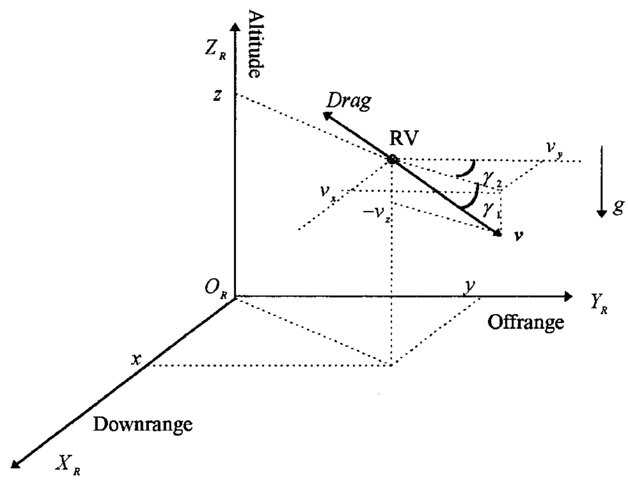

Fig. 8 Estimated unmodeled accelerations from flight data.

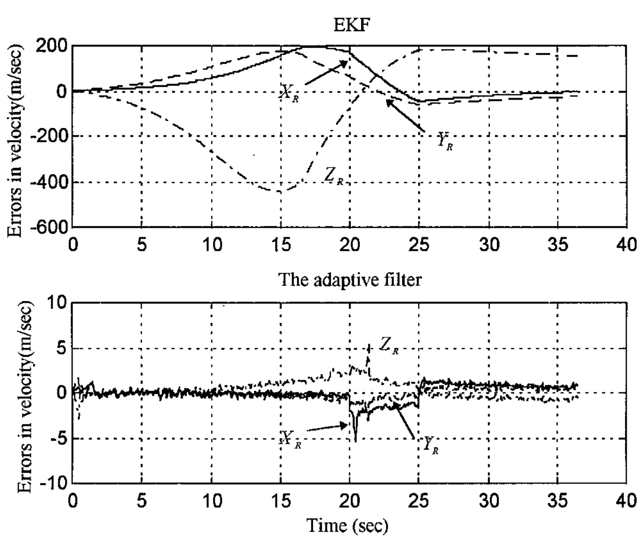

Fig. 7 Estimation errors in velocity.

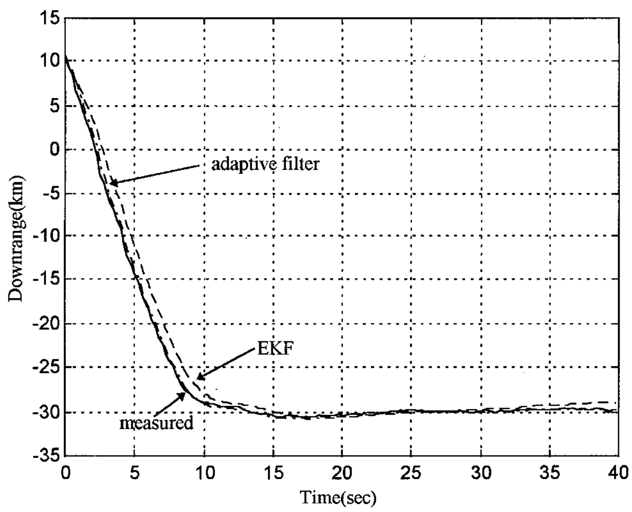

Fig. 9 Trajectory estimation in downrange.

Table 3 Predicted impact point at different prediction times t_p

	$t_p = 10$ s		$t_p = 15$ s		$t_p = 20$ s		
Parameter	Adaptive	EKF	Adaptive	EKF	Adaptive	EKF	Last detection
t_f	45.85	48.5	50.85	48.7	50.8	47.4	50.4
x, km	−12.71	−5.52	−9.32	−5.1	−8.86	−6.37	−9.04
y, km	18.9	−12.55	23.34	11.46	23.58	18.46	23.36

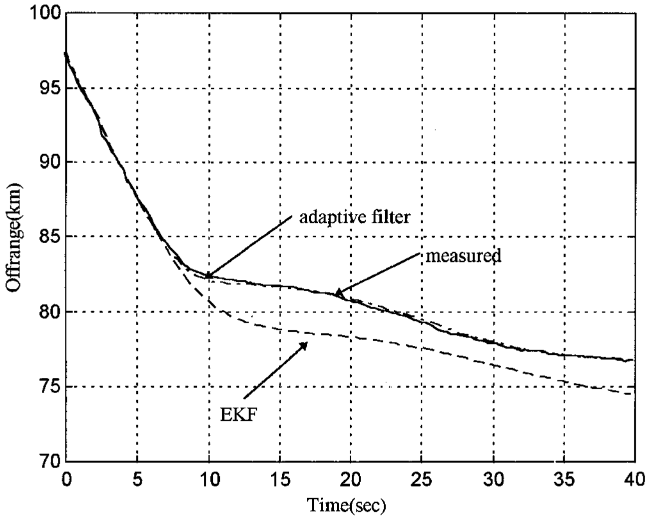

Fig. 10 Trajectory estimation in offrange.

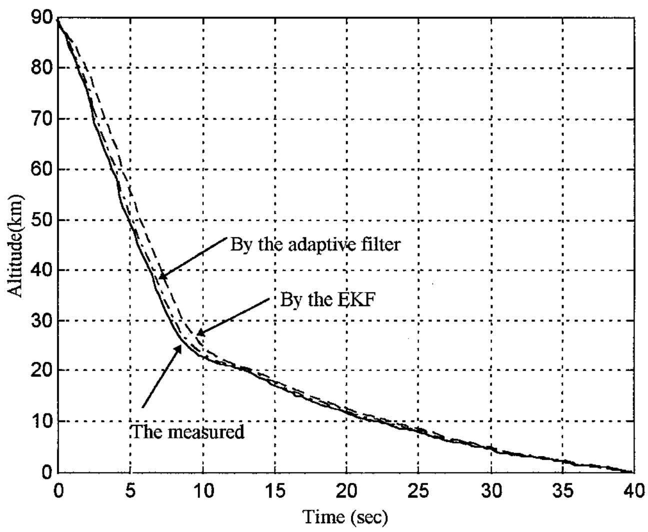

Fig. 11 Trajectory estimation in altitude.

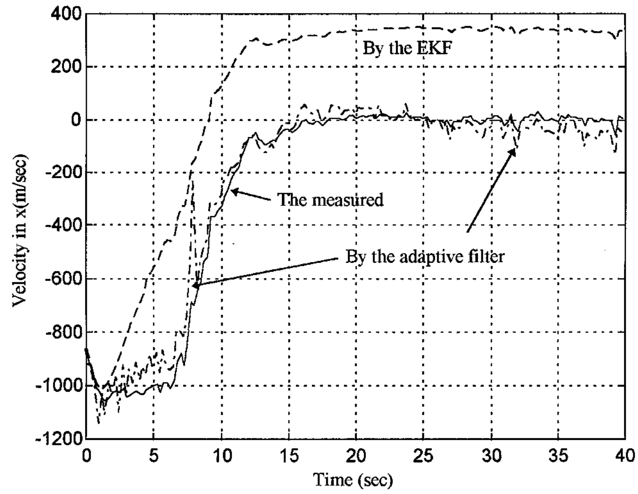

Fig. 12 Velocity estimation in X_R.

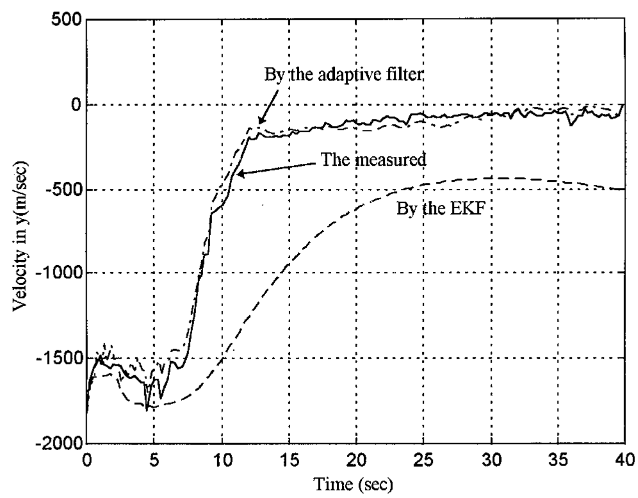

Fig. 13 Velocity estimation in Y_R.

and 24,400 kg/m^2. The rms values for the proposed method indicate lower errors than those obtained with the EKF method.

B. Case 2: Real-Flight Analysis

A flight test was arranged to evaluate a RV system. The RV was launched at ~400 km from the impact point. The raw measured 40-s data span of the RV by a precision radar with a sampling rate of 4 Hz in this flight test is utilized for this study. The RV was first detected by the radar after reentry in the coast phase at a range of 132.5 km with $\gamma_1 = 42.37$ deg and $\gamma_2 = 6.282$ deg with respect to the radar. The first detections are also adopted as the initial conditions and $\beta = 2440$ kg/m^2 during estimation. Figure 8 displays the detected acceleration inputs in three axes and reveals the larger gaps between the real system and the mathematical model.

Figures 9–11 demonstrate the comparison of the measured and the estimated trajectories in downrange, offrange, and altitude, respectively. The filtered trajectory in which the proposed adaptive filter is used follows the measured trajectory well, as opposed to estimation through use of the EKF. The velocity estimation comparison in the three axes are presented in Figs. 12–14 and show that the proposed scheme is better able to keep up with the sensed velocity than the EKF.

Let the prediction time t_p be the starting time after the first detection is received from the radar. Table 3 lists the predicted impact points in position and velocity with flight time t_f at different t_p by means of Eq. (7) with no input vector. The last column represents the predicted impact based on the last detection from the radar. This impact prediction represents the best estimate of the true impact point. Errors in the prediction variables t_f, x, and y by the adaptive filter at $t_p = 20$ s are 0.4 s, 0.18 km, and 0.22 km, respectively, which are superior relative to the predictions from the EKF, whose errors

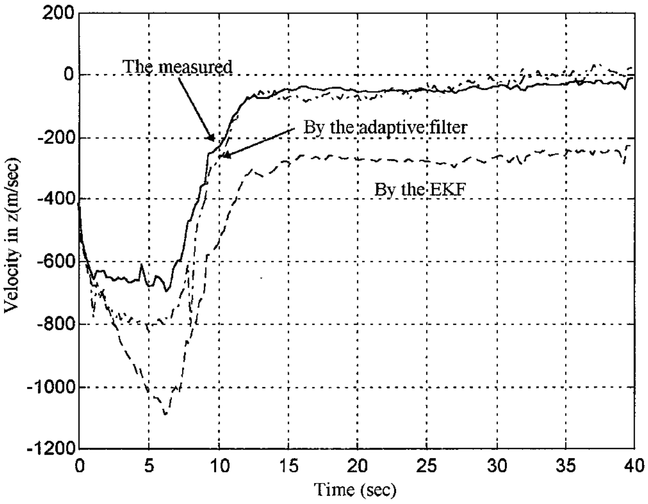

Fig. 14 Velocity estimation in Z_R.

are 3 s, 2.67 km, and 4.9 km, respectively. This table reveals that the predictions from the adaptive filter become stable at $t_p = 15$ s with just 60 samples in each axis, which implies that the adaptive filter is robust with small sample sizes. The important properties provided by the adaptive filter are highly desirable for high-speed target tracking and interception.

V. Conclusions

This study presents a fast and accurate on-line trajectory estimation algorithm for the reentry vehicle. The proposed filter consists of a extended Kalman filter and an input estimator with a detection criterion. It automatically detects all acceleration inputs that are due to modeling errors of the reentry vehicle by sequentially correcting the model to be identified. Numerical simulation with a set of data from a maneuvering reentry vehicle demonstrates that the proposed adaptive filter has much smaller estimation errors than the extended Kalman filter without the input estimator. Robustness with respect to uncertainty in the ballistic coefficient is further validated by simulation. When the adaptive filter is applied to the case of actual measured flight data, it also provides accuracy and robustness in estimation and prediction. The recommended method can support defense and tactical operations for antitactical ballistic missile warfare.

Appendix: Linearization of Matrix $F(x)$

$$\left.\frac{\partial F(X)}{\partial (X)}\right|_{X=\hat{X}_{n/n}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & f_{43} & f_{44} & f_{45} & f_{46} \\ 0 & 0 & f_{53} & f_{54} & f_{55} & f_{56} \\ 0 & 0 & f_{63} & f_{64} & f_{65} & f_{66} \end{bmatrix}_{X=\hat{X}_{n/n}}$$

where

$$f_{43} = \frac{\rho}{60000\beta} g\left(x_4^2 + x_5^2 + x_6^2\right)\cos\gamma_1 \sin\gamma_2 \quad \text{for} \quad x_3 < 30{,}000 \text{ ft}$$

$$= \frac{\rho}{44000\beta} g\left(x_4^2 + x_5^2 + x_6^2\right)\cos\gamma_1 \sin\gamma_2 \quad \text{for} \quad x_3 \geq 30{,}000 \text{ ft}$$

$$f_{44} = -\frac{\rho g}{2\beta}\left[2x_4\cos\gamma_1 \sin\gamma_2 - \frac{x_4 x_6}{\sqrt{x_4^2 + x_5^2}}\sin\gamma_1 \sin\gamma_2 - \frac{\left(x_4^2 + x_5^2 + x_6^2\right)x_5}{x_4^2 + x_5^2}\cos\gamma_1 \cos\gamma_2\right]$$

$$f_{45} = -\frac{\rho g}{2\beta}\left[2x_5\cos\gamma_1 \sin\gamma_2 - \frac{x_5 x_6}{\sqrt{x_4^2 + x_5^2}}\sin\gamma_1 \sin\gamma_2 + \frac{\left(x_4^2 + x_5^2 + x_6^2\right)x_4}{x_4^2 + x_5^2}\cos\gamma_1 \cos\gamma_2\right]$$

$$f_{46} = -\frac{\rho g}{2\beta}\left(2x_6\cos\gamma_1 \cos\gamma_2 + \sqrt{x_4^2 + x_5^2}\sin\gamma_1 \sin\gamma_2\right)$$

$$f_{53} = \frac{\rho}{60000\beta} g\left(x_4^2 + x_5^2 + x_6^2\right)\cos\gamma_1 \cos\gamma_2 \quad \text{for} \quad x_3 < 30{,}000 \text{ ft}$$

$$= \frac{\rho}{44000\beta} g\left(x_4^2 + x_5^2 + x_6^2\right)\cos\gamma_1 \cos\gamma_2 \quad \text{for} \quad x_3 \geq 30{,}000 \text{ ft}$$

$$f_{54} = -\frac{\rho g}{2\beta}\left[2x_4\cos\gamma_1 \cos\gamma_2 - \frac{x_4 x_6}{\sqrt{x_4^2 + x_5^2}}\sin\gamma_1 \cos\gamma_2 + \frac{\left(x_4^2 + x_5^2 + x_6^2\right)x_5}{x_4^2 + x_5^2}\cos\gamma_1 \sin\gamma_2\right]$$

$$f_{55} = -\frac{\rho g}{2\beta}\left[2x_5\cos\gamma_1 \cos\gamma_2 - \frac{x_5 x_6}{\sqrt{x_4^2 + x_5^2}}\sin\gamma_1 \cos\gamma_2 - \frac{\left(x_4^2 + x_5^2 + x_6^2\right)x_4}{x_4^2 + x_5^2}\cos\gamma_1 \sin\gamma_2\right]$$

$$f_{56} = -\frac{\rho g}{2\beta}\left(2x_6\cos\gamma_1 \cos\gamma_2 + \sqrt{x_4^2 + x_5^2}\sin\gamma_1 \cos\gamma_2\right)$$

$$f_{63} = \frac{\rho}{60000\beta} g\left(x_4^2 + x_5^2 + x_6^2\right)\sin\gamma_1 \quad \text{for} \quad x_3 < 30{,}000 \text{ ft}$$

$$= \frac{\rho}{44000\beta} g\left(x_4^2 + x_5^2 + x_6^2\right)\sin\gamma_1 \quad \text{for} \quad x_3 \geq 30{,}000 \text{ ft}$$

$$f_{64} = \frac{\rho g}{2\beta}\left(2x_4\sin\gamma_1 + \frac{x_4 x_6}{\sqrt{x_4^2 + x_5^2}}\cos\gamma_1\right)$$

$$f_{65} = \frac{\rho g}{2\beta}\left(2x_5\sin\gamma_1 + \frac{x_5 x_6}{\sqrt{x_4^2 + x_5^2}}\cos\gamma_1\right)$$

$$f_{66} = \frac{\rho g}{2\beta}\left(2x_6\sin\gamma_1 - \sqrt{x_4^2 + x_5^2}\cos\gamma_1\right)$$

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